Equation of Continuity (Conservation of Charge):

If the total charge crossing a surface bounding a close volume is not zero, then the net charge density within the volume must change with time in such a manner that the time rate of increase of charge within the volume equal to the net rate of flow of charge into the volume. This statement of conservation of charge in a medium is expressed in a mathematical equation called equation of continuity.

Let S is a surface enclosing a volume V and dS ia a small area element on this surface having direction along outward drawn normal. If **J** is current density i. e ., current per unit area normal to direction of current flow at a point on surface element. dS. Then J dS represent the charge per unit time leaving the volume V through dS. Therefore the time rate at which charge leaves the volume V bounded by entire surface S is given by

$$\oint\!\!\oint_S JdS$$

If q is charge contained in volume V, then according to charge conservation law the above integral must be equal to –dq/dt, where dq/dt is the rate of flow of charge into volume V.

$$\oint\!\!\oint_S JdS = -\frac{dq}{dt}$$

But the charge q can be written in terms of the charge density ρ as

$$Q = \iiint_V \rho dV$$

$$\therefore \iint_S JdS = -\frac{d}{dt}\iiint_V \rho dV$$

or $$\iint_S JdS = -\iiint_V \frac{d\rho}{dt} dV$$

changing surface integral into volume integral using Gauss divergence theorem we can write

$$\iiint_V DivJdV = -\iiint_V \frac{d\rho}{dt} dV$$

or $$\iiint_V DivJdV + \iiint_V \frac{d\rho}{dt} dV = 0$$

$$\iiint_V \left(DivJ + \frac{d\rho}{dt} \right) dV = 0 \Rightarrow DivJ + \frac{d\rho}{dt} = 0$$

$$\Rightarrow DivJ + \frac{d\rho}{dt} = 0$$

which is the required equation of continuity and expresses the charge conservation law.

The current is called steady state or stationary current if there is no accumulation of charges at any point in the region i.e., for stationary current flow dρ/dt=0 at any point in the region. Therefore criterian for stationary current flow is

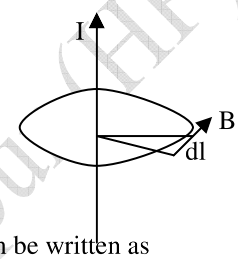
$$DivJ = \nabla.J = 0$$

Ampere's Law :

The line integral of the magnetic induction B over a close path is

$$\oint B.dl = \oint Bdl\cos\theta = \oint Bdl = \oint \frac{\mu I}{2\pi r} r d\theta = \frac{\mu I}{2\pi}\oint d\theta = \frac{\mu I}{2\pi}(2\pi) = \mu I$$

$$\oint B.dl = \mu I$$



It is know as Ampere's circuital law. In term of the magnetic field (as B = μH) it can be written as

$$\oint H.dl = I$$

Maxwell's Postulate (Concept of displacement current):

Ampere's law in circuital form is expressed as

$\oint \vec{H}.d\vec{l} = I$, which in term of charge density can be written as

$$\oint \vec{H}.d\vec{l} = \int \vec{J}.d\vec{S}$$

Changin the surface integral on LHS to volume integral using Stoke's theorem we have

$$\oint_S Curl\vec{H} = \oint \vec{J}.d\vec{S}$$

or $$\oint \left(Curl\vec{H} - \vec{J}\right)dS = 0 \Rightarrow Curl\vec{H} - \vec{J} = 0 \Rightarrow Curl\vec{H} = \vec{J}$$

Since divergence of curl of a vector is always zero, therefore Div(Curl H) =0 implies

$$Div\vec{J} = 0$$

But divergence of J is only zero if dρ/dt =0 i..e., if the charge density is static and the current is stationary. It means the Ampere's law given by equation (1) is only valid for stationary currents and is insufficient for the case of time varying field.

Maxwell postulated how one can modify Ampere's law so as to make it consistent with the equation of continuity. Maxwell assumed that the definition of currnt density J is incomplete and something, say J_d , must be added to it so that the current density becomes solenoidal. Thus the total solenoidal current having its divergence zero becomes $J+J_d$ and consequently the Ampere's law takes the form

$$Curl\vec{H} = \vec{J} + \vec{J}_d$$

In order to identify J_d let us take divergence of this equation i.e.,

$$Div\left(Curl\vec{H}\right) = Div(J + J$$

or $$0 = DivJ + DivJ_d$$

or $$DivJ_d = -DivJ = -(-\frac{d\rho}{dt}) = \frac{d\rho}{dt}$$

From Gauss's law in dielectric DivD = ρ, therefore we have

$$DivJ_d = \frac{d}{dt}\left(Div\vec{D}\right) = Div\left(\frac{d\vec{D}}{dt}\right)$$

or $$\vec{J}_d = \frac{d\vec{D}}{dt}$$

Therefore the modified Ampere's law become

$$Curl\vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

The term added to Ampere's law to include time varying currents is known as displacement current, because it arises when electric displacement vector D changes with time. By adding this term Maxwell found that displacement current is as effective as the conduction current J for producing magnetic fields.

Characteristics of displacement current:

1. Displacement current is a current only in the sense that it produces magnetic field. It has non of the other properties of the current since it is not linked with the motion of free charges. For example displacement current has finite value even in perfect vacuum where there are no charges at all.
2. The magnitude of the displacement current is equal to rate of change of electric displacement vector i.e., $J_d = dD/dt$.
3. Displacement current serves the purposes to make the total current continuous across the discontinuity in the conduction current. As an example, a battery charging a capacitor makes a closed current loop in terms of total current $J+J_d$.
4. Displacement current in a good conductor is negligible as compared to conduction current at any frequency less than optical frequencies (10^{15} Hz).