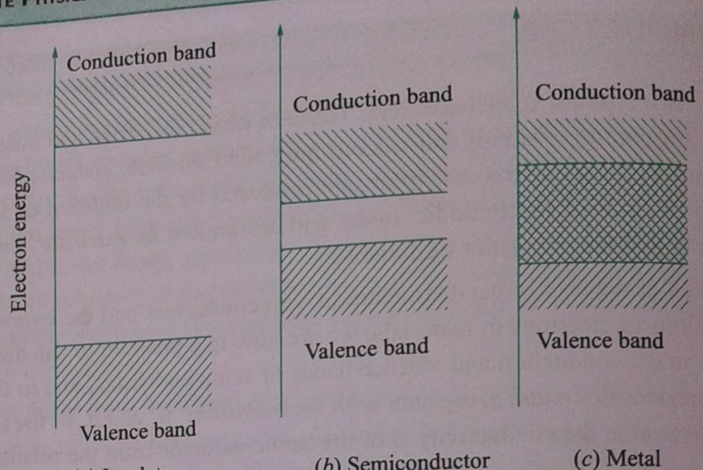


(*a*) Insulator (*b*) Semiconductor (*c*) Metal

Fig. 10.6 Typical energy bands

IV. INTRINSIC SEMICONDUCTORS

These are semiconductors which have no impurity type of imperfection but may have defects. Germanium and silicon in the pure form are two examples of intrinsic semiconductors. In these cases, there are four nearest neighbours to each atom and there are two electron bonds between each of the neighbours and the atom. In an intrinsic semiconductor a forbidden gap exists between a filled valence band and the conduction band, but the width of the forbidden gap expressed in energy is much smaller than that of an insulator.

Conduction occurs in an intrinsic semiconductor by thermal excitation of electrons from the valence to the conduction band. Electrons excited to the higher, predominantly empty conduction band levels can move from atom to atom, thus contributing to conduction when a field is applied to the semiconductor.

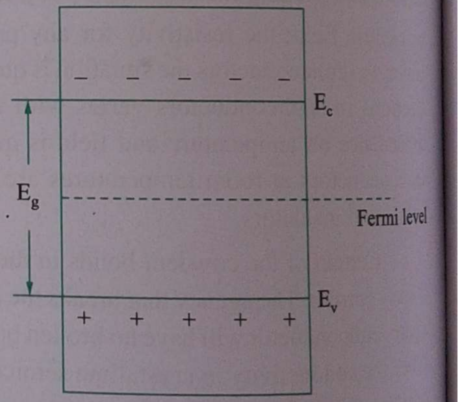


Fig. 10.7 Thermal excitation of electrons in intrinsic semiconductors

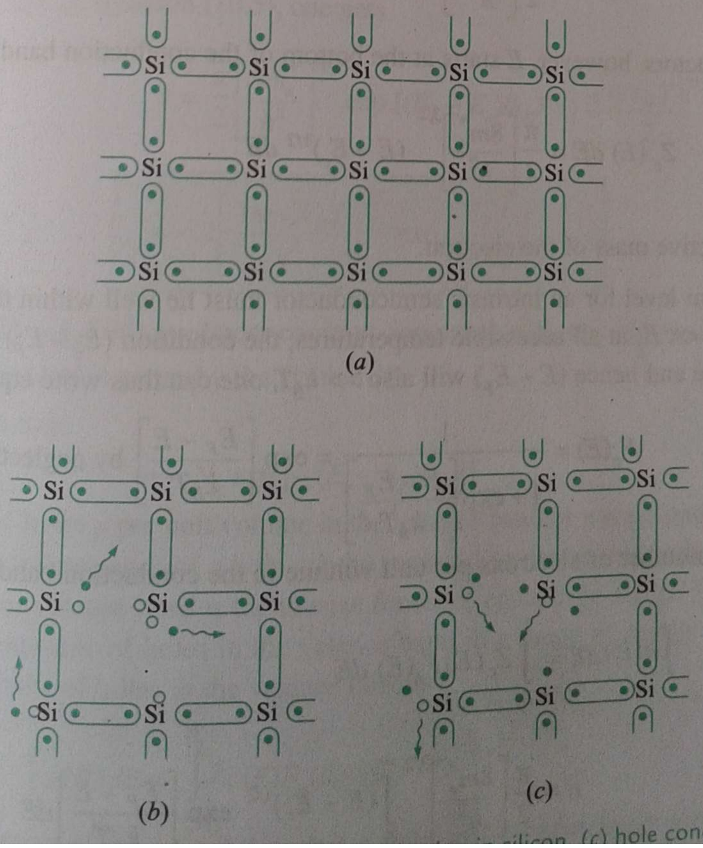
V. CONDUCTIVITY AND TEMPERATURE

The conductivity of a semiconductor depends strongly on temperature because of the need for thermal energy to excite electrons into the conduction band. The higher the temperature of the semiconductor, the greater the number of electrons excited. Therefore, in a semiconductor the higher the temperature, the lower the resistivity. Note that this is just the reverse in metals.

It might be thought a threshold temperature also exists for *intrinsic activation* of electrons as threshold frequency for photoconductivity. This is not the case. The process of thermal activation of electrons is somewhat similar to that of heating a liquid to the boiling point, in that thermal energy *evaporates* electrons from the valence band to the conduction band at any temperature above absolute zero. The rate of evaporation depends on temperature. However, thermal activation is different from boiling, in that there is no specific temperature at which the rate of *evaporation* increases drastically, and hence no real threshold exists. Temperatures much lower than that corresponding to the width of the forbidden gap are sufficient to excite a substantial number of electrons from the valence band to the conduction band, just as many water molecules can evaporate from a heated body of water even though the liquid is well below the boiling point.

VI. STATISTICS OF ELECTRONS AND HOLES IN INTRINSIC SEMICONDUCTORS

As previously stated, at any finite temperature some electrons will acquire sufficient thermal energy to raise them from the valence band to the conduction band. The actual number depends on the number of permissible electron energy levels and the probability of these levels being occupied.



(*a*)

(*b*)

(*c*)

Fig. 10

ilicon (c) hole conduction in silicon

In order to calculate the intrinsic carrier concentration, we first calculate the number of electrons excited into the conductions band at any temperature T kelvin and which in turn are free to migrate in the crystal. In carrying out these calculations it will be assumed that the electrons in the conduction band behave as if they are free with *effective mass* m_e^* and energy will be measured from the top of the valence band.

Under the conditions of thermal equilibrium, the number of electrons per unit volume having energy in a range dE about E in the conduction band of an intrinsic semiconductor is,

$$n(E)\,dE = Z_c\,(E)\,dE\,F_n(E)$$

where $F_n(E)$ represents the equilibrium *Fermi distribution* function and Z_c (E) is the *density of states factor*. The occupation number for electron is

$$F_n(E) = \frac{1}{1+\exp\left[\dfrac{E-E_F}{k_BT}\right]} \tag{10.1}$$

and

$$Z_c(E) = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} (E-0)^{1/2} \text{ for metals} \tag{10.2}$$

For semiconductors, however, E starts at the bottom of the conduction band, hence

$$Z_c\,(E)\,dE = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} (E-E_c)^{1/2}\,dE \tag{10.3}$$

where, m_e^* is the effective mass of the electron.

Since the Fermi level for an intrinsic semiconductor must lie well within the forbidden energy region, and since $k_BT << E_g$ at all accessible temperatures, the condition $(E_c - E_F) >> k_BT$ will always be satisfied in practice and hence $(E - E_F)$ will also $>> k_BT$, one can thus write equation (10.1) as

$$F_n(E) = \frac{1}{1+\exp\left[\dfrac{E-E_F}{k_BT}\right]} = \exp\left[\frac{E_F-E}{k_BT}\right] \text{ by neglecting the term 1}$$

Thus the total number of electrons per unit volume in the conduction band is

$$\int n(E)\,dE = \int_{E_c}^{\infty} Z_c(E)F_n(E)\,dE$$

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} \exp\left[\frac{E_F-E}{k_BT}\right] dE \tag{10.4}$$

Let

$$E - E_c = x$$

Equation (10.4) becomes

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} \exp\,(E_F/k_BT) \int_0^\infty x^{1/2} \exp\,[-(E_c + x)/k_BT]\,dx$$

i.e.,

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} \exp\,[(E_F - E_c)/k_BT] \int_0^\infty x^{1/2} \exp\,(-\alpha x)\,dx \tag{10.5}$$

with $\alpha = \dfrac{1}{k_BT}$

It can be shown with the help of gamma functions that

$$\int_0^\infty x^{1/2} e^{-\alpha x}\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{1}{\alpha^{3/2}}\right)$$

$$= \frac{\sqrt{\pi}}{2}[(k_BT)^{3/2}]$$

Substituting this in equation (10.5), one gets

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} \exp\,[(E_F - E_c)/k_BT]\left[\frac{\sqrt{\pi}(k_BT)^{3/2}}{2}\right]$$

or

$$\boxed{n = 2\left[\frac{2\pi m_e^* k_BT}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_c}{k_BT}\right]} \tag{10.6}$$

The term $2(2\pi m_e^* k_BT/h^2)$ is almost constant compared with the exponential term, and so it is called *pseudo constant* and is given by the symbol N_c.

So finally, we have

$$n = N_c \exp\,[-(E_c - E_F)/k_BT] \tag{10.7}$$

The number of holes p per unit volume in the valence band of a semiconductor can be found in a similar manner. As the presence of a hole can be regarded as the absence of an electron, the Fermi function of holes in the valence band is the inverse function, equal to $[1 - F_n(E)]$. In this case we may write $p(E)\,dE$ as the number of holes in the valence band in a range of energy dE about E; then the expression for the density of holes in the valence band is

$$\int p(E)\,dE = \int_{-\infty}^{E_v} Z_v(E)F_p(E)\,dE \tag{10.8}$$

The quantity $F_p(E)$ is the probability that a hole will be associated with a quantum state of energy E, in other words, that such a state will be unoccupied by an electron. This probability is just one minus the probability of [illegible] $F_n(E)$ is given by equation (10.1), it is easily seen that

$$F_p(E) = [1 - F_n(E)] = 1 - \frac{1}{1 + \exp[(E - E_F)/k_BT]} = \frac{1 + \exp[(E - E_F)/k_BT] - 1}{1 + \exp[(E - E_F)/k_BT]}$$

$$= \frac{1}{1 + \exp[(E_F - E)/k_BT]} \tag{10.9}$$

Again, if the Fermi energy is several k_BT units above the edge of the valence band, that is, if $(E_F - E_v) >> k_BT$, the exponential factor in the denominator of equation (10.9) will be much larger than unity for all values of E in the valence band, whereby one may set

$$F_p(E) = \exp\left[-\frac{(E_F - E)}{k_BT}\right] \tag{10.10}$$

Referring equation (10.3), one can write

$$Z_v(E) = \frac{\pi}{2}\left[\frac{8m_p^*}{h^2}\right]^{3/2} (E_v - E)^{1/2} \tag{10.11}$$

where, m_p^* is the effective mass of the hole.

Using relations (10.10) and (10.11) and integrating over the valence band, equation (10.8) becomes

$$p = \frac{\pi}{2}\left[\frac{8m_p^*}{h^2}\right]^{3/2} \int_{-\infty}^{E_v} (E_v - E)^{1/2} \exp\left[\frac{E - E_F}{k_BT}\right] dE \tag{10.12}$$

Let $E_v - E = x$

Equation (10.12) becomes

$$p = \frac{\pi}{2}\left[\frac{8m_p^*}{h^2}\right]^{3/2} \exp(-E_F/k_BT) \int_0^{\infty} x^{1/2} \exp[(E_v - x)/k_BT]\, dx$$

Let $\alpha = \frac{1}{k_BT}$

Now $p = \frac{\pi}{2}\left[\frac{8m_p^*}{h^2}\right]^{3/2} \exp[(E_v - E_F)/k_BT] \int_0^{\infty} x^{1/2} \exp(-\alpha x)\, dx$

This integral here is similar to the integral of equation (10.5). Hence the expression for the density of holes becomes

$$p = 2\left[\frac{2\pi m_p^* k_BT}{h^2}\right]^{3/2} \exp[(E_v - E_F)/k_BT] = N_v \exp[(E_v - E_F)/k_BT]$$

with $N_v \text{ (or } N_p) = 2\left[\frac{2\pi m_p^* k_BT}{h^2}\right]^{3/2}$ (10.13)

This is called pseudo-constant.

For intrinsic semiconductor, $n = p$

Therefore

$$\boxed{(m_e^*)^{3/2} \exp[-(E_c - E_F)/k_BT] = (m_p^*)^{3/2} \exp[-(E_F - E_v)/k_BT]} \tag{10.14}$$

$$\frac{\exp\left(\frac{E_F - E_c}{k_B T}\right)}{\exp\left(\frac{E_v - E_F}{k_B T}\right)} = \left[\frac{m_p^*}{m_e^*}\right]^{3/2}$$

$$\exp\left[\frac{E_F + E_F - (E_c + E_v)}{k_B T}\right] = \left[\frac{m_p^*}{m_e^*}\right]^{3/2}$$

$$\frac{2E_F}{k_B T} = \frac{E_c + E_v}{k_B T} + \frac{3}{2}\ln\left[\frac{m_p^*}{m_e^*}\right]$$

$$E_F = \frac{E_c + E_v}{2} + \left[\frac{3k_B T}{4}\right]\ln\left[\frac{m_p^*}{m_e^*}\right] \tag{10.15}$$

If it happens that m_p^* equals m_e^*, then

$$\boxed{E_F = \frac{E_c + E_v}{2}}$$

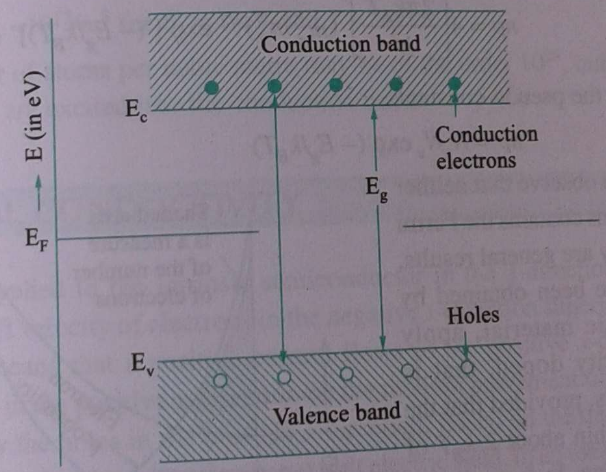


Fig. 10.8(A) Generation of electron-hole pair in an intrinsic semiconductor

that is, the Fermi level is mid-way between the valence band and conduction band, but normally m_p^* is greater than m_e^*, so E_F is just above the middle, and rises slightly with increasing temperature. The different results are shown graphically in Fig. 10.9.

Equation (10.15) can be re-written in terms of the pseudo-constants, as it can be seen that

$$\frac{N_c}{N_v} = \left[\frac{m_e^*}{m_p^*}\right]^{3/2}$$

Therefore
$$E_F = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left[\frac{N_v}{N_c}\right] \tag{10.16}$$

From equation (10.15), it can be concluded that if $m_p^* \neq m_e^*$, the Fermi level is a function of temperature.

For a given semiconducting substance, the effective masses and the energy gap E_g are fixed; hence the product np in a given material must be a function of temperature only. This is essentially a *mass-action* law governing the relative concentrations of holes and electrons in a given material. If the semiconductor is in the pure or intrinsic state, then the concentrations of holes and electrons must be equal, as we have already seen, because the holes and electrons which can be present are those which are generated in pairs by direct thermal excitation of valence band electrons. In such a material we take the product of the expressions for electrons and holes, equations (10.6) and (10.13) we get

$$np = 4\left[\frac{2\pi k_B T}{h^2}\right]^3 (m_e^* m_p^*)^{3/2} \exp\left[-(E_c - E_v)/k_B T\right]$$

Now, $(E_c - E_v)$ equals E_g, the width of the *forbidden gap*. Therefore we can write

$$np = 4\left[\frac{2\pi k_B T}{h^2}\right]^3 (m_e^* m_p^*)^{3/2} \exp\left[(-E_g/k_B T)\right] \tag{10.17}$$

or, alternatively, using the pseudo-constants,

$$np = N_c N_v \exp(-E_g/k_B T) \tag{10.18}$$

It is important to observe that neither of these last two equations contains the Fermi level, and therefore they are general results, which, whilst they have been obtained by considering an intrinsic material, apply equally well to impurity doped, that is extrinsic, semiconductors, provided that the Fermi level is never within about 2 $k_B T$ of either forbidden band edge. In fact, all that happens is that if an intrinsic material has impurities added so that the Fermi level rises, then n becomes greater than p, and if the Fermi level drops below the intrinsic level, p exceeds n. All the time, however, the product np is constant at constant temperature.

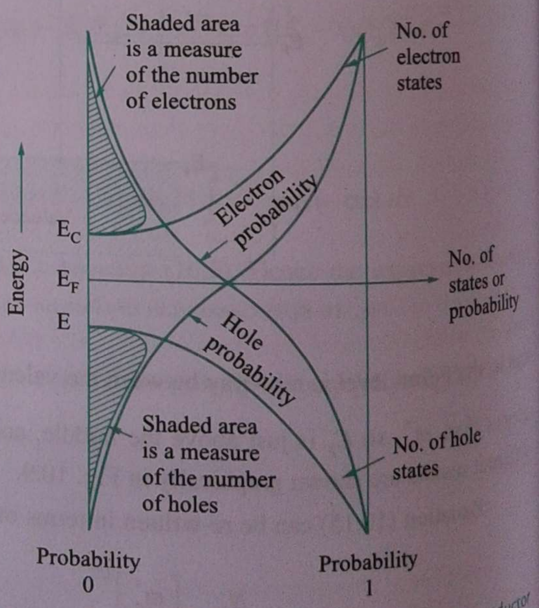


Fig. 10.9 Electrons and holes in an intrinsic semiconductor

Returning to the case of the *intrinsic semiconductor*, the density of electrons equals the density of holes, and they are both called the *intrinsic density*, n_i, where

$$n_i^2 = np = (N_cN_v)\exp(-E_g/k_BT)$$

i.e.,

$$n_i = (np)^{1/2} = (N_cN_v)^{1/2}\exp(-E_g/2k_BT) \quad (10.18a)$$

Substituting for N_cN_v and simplifying,

$$\boxed{n_i = 2\left[\frac{2\pi k_BT}{h^2}\right]^{3/2}(m_e^* m_p^*)^{3/4}\exp(-E_g/2k_BT)} \quad (10.19)$$

If $m_e^* = m_p^* = m$, where, m is the rest mass of the electron, then equation (10.19) becomes,

$$n_i = 2\left[\frac{2\pi mk_BT}{h^2}\right]^{3/2}\exp(-E_g/2k_BT) \quad (10.20)$$

or

$$n_i = CT^{3/2}\exp[-E_g/2k_BT] \quad (10.21)$$

with

$$C = 2\left[\frac{2\pi mk_B}{h^2}\right]^{3/2} = \left[\frac{2\pi \times 9.1\times10^{-31}\times1.38\times10^{-23}}{(6.626)^2\times10^{-68}}\right]^{3/2} \quad (10.22)$$

i.e.,

$$C = 4.83\times10^{21}. \text{ Hence } n_i = 4.83\times10^{21}\times T^{3/2}\exp(-E_g/2k_BT) \quad (10.23)$$

or

$$n_i \approx 10^{17}/m^3 \text{ for } E_g = 1 \text{ eV and } T = 300 \text{ K}$$

Since the number of atoms per cubic metre may be of the order 10^{28}, only a very small fraction of the valence electrons are excited into the conduction band.

VII. ELECTRICAL CONDUCTIVITY

If an electric field is applied to the intrinsic semiconductor in the x-direction, the electrons receive energy and there is a drift velocity of electrons in the negative x-direction superimposed on their random thermal motion. This means that a positive current flows in the positive x-direction. The field also imposes a drift velocity in the positive x-direction on the random movement of the holes. Thus positive charge is also carried by the holes in the positive x-direction. Based on the definition of drift velocity; the *mobility* μ can be defined as the drift velocity per unit electric field. In an ideal intrinsic semiconductor the mobility is determined by scattering of the electrons by lattice vibrations. At low enough temperatures, however, where the *mean free path* of the electrons becomes long, scattering by impurity atoms and defects sets a limit to the mobility attained.

Since there are two types of carriers in the intrinsic semiconductor, electrons and holes, its *specific conductance* is sum of the conductivities σ_n due to free electrons and σ_p due to holes. Thus the electric conductivity of an intrinsic semiconductor is

$$\sigma_i = \sigma_n + \sigma_p = ne\,\mu_e + pe\,\mu_p \quad (10.24)$$

Since for $n = n_i$, one gets

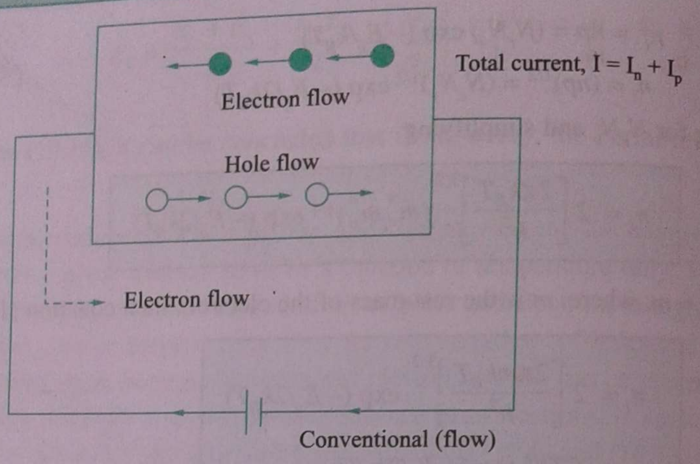


Fig. 10.10 Conduction of electric current in an intrinsic semiconductor

The mobilities depend on temperature as a modest power law. *i.e.,* μ goes as $\frac{l}{T^{3/2}}$

or
$$\mu_e = \alpha\, T^{-1.5} \quad \text{and} \quad \mu_p = \beta\, T^{-1.5}$$

Thus, $\mu_e + \mu_p = (\alpha + \beta)T^{-1.5} = \gamma\, T^{-1.5}$

Equation (10.24) becomes

$$\sigma_i = \gamma\,(en_i)T^{-3/2}$$

Substituting for n_i from equation (10.21), one gets

$$\sigma_i = \gamma\, C\, e \exp\,(-E_g/2k_BT)$$

$$\sigma_i = B \exp\,(-E_g/2k_BT) \qquad (10.25)$$

or

$$\rho_i = \frac{\exp(E_g/2k_BT)}{B}$$

i.e.,

$$\rho_i = A \exp\left[\frac{E_g}{2k_BT}\right] \qquad (10.26)$$

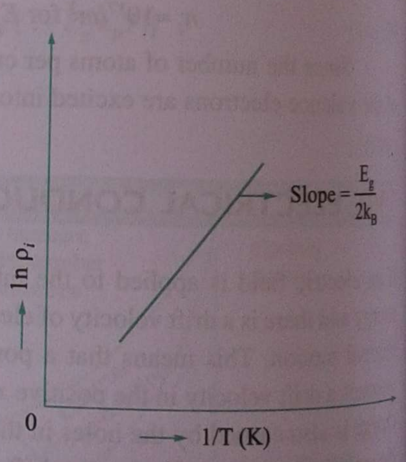


Fig. 10.11 ln ρ_i versus 1/T

Taking logarithms on both sides,

$$\boxed{\ln \rho_i = \frac{E_g}{2k_BT} + \ln A} \qquad (10.27)$$

Equation (10.27) suggests us a method of determining the *energy gap* of an intrinsic material.

A measurement of the intrinsic resistivity of the material at various temperatures helps us to plot the results on a semilogarithmic paper. The slope of the curve is ($E_g/2k_B$), and hence the energy gap can be determined. See Figs. 10.11 and 10.12.

rate of change must be equal to the acceleration due to the field, or $eE/m = v_d/\tau = \mu E/\tau$. Therefore, $\mu = e\tau/m$. The parameter τ is called *relaxation time*.

In a perfect crystal, the conduction electrons move in an ideally triply periodic potential field. In order to explain the origin of collisions it becomes necessary to postulate that deviations from ideally perfect lattice occur in crystals. There are several kinds of imperfections including *substitutional* and *interstitial atoms*, *missing atoms* or *vacancies*, *dislocations* etc. In addition to these built-in defects the thermal vibrations of atoms at temperatures above absolute zero produce instantaneous displacements which also disrupt the ideal periodicity in a crystal. All these imperfections disrupt the periodicity of atomic array and cause the electrons to be scattered. The two important causes of scattering in semiconductors are (*i*) atomic vibrations (phonons) (*ii*) ionised impurity atoms. The effect that these imperfections have on the mobilities of electrons or holes can be determined most easily by calculating the corresponding relaxation time. In performing such calculations it is necessary to consider the energy values that the electrons can have and it is convenient to average the relaxation times over the entire energy distribution. An actual calculation of the mobility μ_L due to scattering by phonons is the $T^{-3/2}$ temperature dependence. Similarly the mobility due to ionised impurities scattering μ_I is $T^{3/2}$ temperature dependence. The actual mobility is given by

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

i.e.,

$$\boxed{\frac{1}{\mu} = \frac{a}{T^{3/2}} + bT^{3/2}} \qquad (10.57)$$

Drift and Diffusion

The current that flows across a semiconducting crystal has two components:

1. *Drift current* 2. *Diffusion current.*

1. *Drift Current*

In a perfect (defect free) crystal the periodic electric field enables electrons and holes to move freely as if in a vacuum. In this case the wave model of the electron is more appropriate than the particle model. The defects in real crystals cause the periodic electric field to be disturbed and, for $T > 0$ K, thermal energy can be considered to cause the lattice ions to vibrate. These imperfections cause scattering of the free carriers resulting in finite conductivity.

By considering the free electrons in a semiconductor to be analogous to the molecules in a low pressure gas, the kinetic energy of a free electron due to its thermal energy is $1.5\ k_B T$ from which its mean (thermal) velocity can be estimated to be of the order of 10^5 m/s. In the absence of any externally applied electric field, the random motion of free carriers within a crystal does not result in a net transfer of charge since charge movement in any direction is balanced by charge movement in any other direction. When a voltage is applied across the material, each carrier experiences a force attracting it to one end of the material, electrons being attracted to the positive potential due to their negative charge and holes to the negative potential. This net movement in charge carriers, termed *drift* is superimposed on the random *thermal movement* and results in current flow through the crystal. The *electric current density* is given by

$$J = nev_d$$

where v_d is the drift velocity. Since the usual statement of Ohm's law is $J = \sigma F$, where σ is the electrical conductivity of the crystal, this conductivity is

$$\sigma = J/E = (nev_d/E)$$

$$\sigma = (ne/E)\,(eE\tau/m) \text{ with } v_d = (eE\tau/m)$$

i.e.,

$$\sigma = (ne^2\tau/m)$$

i.e.,

where τ is the *relaxation time* between collisions (10.58)

Again

$$v_d \propto E$$

$$v_d = \mu E$$

where, the parameter μ is the mobility of charge carriers within the crystal

Thus,

$$J = nev_d = ne\mu E$$

$$\sigma = J/E = ne\mu$$

or

or in terms of resistivity ρ (Ω-m),

$$\rho = 1/\sigma = (1/ne\mu) \tag{10.59}$$

If the material is a semiconductor, the current flow would be due to electron and hole movement. Correspondingly the current densities due to electron drift and hole drift are:

$$J_n\,(drift) = n\,\mu_n\,eE \tag{10.60}$$

$$J_p\,(drift) = p\,\mu_p\,eE \tag{10.61}$$

respectively. Although these charge movements are in opposite directions, the electron movement is negative charge movement while the hole component is positive charge movement and so the total drift current is the sum of the two components, thus

$$J\ (drift) = J_n\,(drift) + J_P\,(drift)$$

i.e.,

$$J\ (drift) = ne\mu_n E + pe\,\mu_p E \tag{10.62}$$

Comparison with $J = \sigma E$ shows that the conductivity of a semiconductor is given by

$$\sigma = ne\,\mu_n + p\,e\mu_p$$

For an intrinsic semiconductor, $n = p = n_i$ and so the intrinsic conductivity is given by

$$\boxed{\sigma_i = n_i e\,(\mu_n + \mu_p)} \tag{10.63}$$

For extrinsic semiconductors with typical dopant levels, the minority carrier component is negligible at normal operating temperatures.

Conductivity is a function of temperature. If an increase of temperature causes an increase in free charge, the conductivity of the material is increased. However, temperature increase also causes increased vibration of the lattice ions which causes increased scattering, reduced carrier mobility and hence reduced conductivity. The variation of conductivity of a material with temperature depends on which of these effects dominates. For a conductor there is not a significant increase in free charge with increase of temperature and therefore the increased scattering causes the conductivity of a conductor to fall as the temperature increases. Alternatively, the free carrier density of an intrinsic semiconductor increases rapidly with temperature resulting in increased conductivity. A conductor is said to have a *positive temperature coefficient resistance* since its resistance increases with temperature while an intrinsic semiconductor has a *negative coefficient.*

At normal operating temperatures an extrinsic semiconductor has properties similar to those of a conductor. An increase of temperature does not significantly change the free carrier density since most of the dopant atoms are already ionised; thus increased scattering causes conductivity to fall. However, as the transition temperature is approached, the generation of electron-hole pairs does cause the free carrier density to increase significantly and the material takes on intrinsic properties. Thus an extrinsic semiconductor has a positive temperature coefficient of resistance at normal temperatures changing to a negative coefficient near the transition temperature.

2. *Diffusion Current*

In addition to the drift motion of the carriers under the influence of an electric field the carriers in semiconductors may move by diffusion. Diffusion occurs whenever there is a non-uniform concentration of charge carriers at some places of the crystal.

The free electron density in a homogeneous semiconductor in the absence of an applied electric field is uniform at equilibrium due to a combination of the *random thermal motion* of the electrons and the repulsive force between them caused by their like charge. If excess carriers are introduced locally within such a semiconductor, either by causing carrier generation by heating or incident radiation or by injecting carriers into the material via a surface contact, a non-uniform distribution is created. Ignoring the process of recombination which is an additional phenomenon, the excess carriers move from the region of higher density to regions of lower density tending to produce a uniform distribution. The transport mechanism is called *diffusion* and it take place in addition to *drift* caused by an applied electric field.

Let us suppose that the concentration Δn of electrons varies with distance x in the semiconductor, the concentration of gradient being

$$\frac{\partial(\Delta n)}{\partial x}$$

Partial derivatives are used here as Δn is a function of both time and distance. Fick's law states that the rate at which carriers diffuse is proportional to the density gradient and the movement is in the direction of negative gradient, the rate of flow of electrons is proportional to

$$-\frac{\partial(\Delta n)}{\partial x}$$

from which the rate of flow across unit area is got, equal to

$$-D_n\frac{\partial(\Delta n)}{\partial x} \tag{10.63a}$$

where D_n is the *diffusion coefficient* for electrons in the semiconductor concerned. Partial derivatives are used in equation (10.63*a*) as Δn is a function of both time and distance. This flow constitutes an electron diffusion current density and since conventional current is the rate of positive charge

$$J_n \text{ (diffusion)} = -(e) \text{ (rate of flow across unit area)}$$

$$= eD_n = \frac{\partial(\Delta n)}{\partial x} \tag{10.64}$$

If an excess hole concentration is created in the same region, hole diffusion takes place in the same direction at a rate per unit area

$$= -D_p\frac{\partial(\Delta p)}{\partial x}$$

resulting in a *hole diffusion current density*

J_p (diffusion) = + (e) (rate of flow across unit area)

$$= -e\,D_p \frac{\partial(\Delta p)}{\partial x} \tag{10.65}$$

where D_p is the *hole diffusion coefficient*.

If there is an electric field E and a concentration gradient in the x-direction, the total hole-current is the sum of the drift current $(pe)\mu_p$ and the diffusing current given by equation (10.65).

Thus,
$$J_p = J_p\,(\text{drift}) + J_p\,(\text{diffusion})$$

$$J_p = p\,e\mu_p - eD_p \frac{\partial(\Delta p)}{\partial x} \tag{10.66}$$

Similarly for the electrons, the total current density

$$J_n = ne\mu_n + eD_n \frac{\partial(\Delta n)}{\partial x} \tag{10.67}$$

Einstein Relation

Although the carrier transport mechanisms drift and diffusion have been considered independently, they are both dependent on the scattering processes responsible for hindering the flow of carriers in a semiconductor. Einstein showed that the parameters describing the two processes, mobility μ and diffusion coefficient D respectively, are directly related. At equilibrium with no applied electric field, the free electron distribution is uniform and there is no net current flow. Any tendency to disturb the state of equilibrium which would lead to a diffusion current, creates an internal electric field and a drift current balancing the diffusion component. Under equilibrium conditions, therefore, the drift and diffusion currents due to an excess density of electrons, and from equations (10.60) and (10.63*a*)

$$(\Delta n)eE\mu_n = eD_n \frac{\partial(\Delta n)}{\partial x} \tag{10.67a}$$

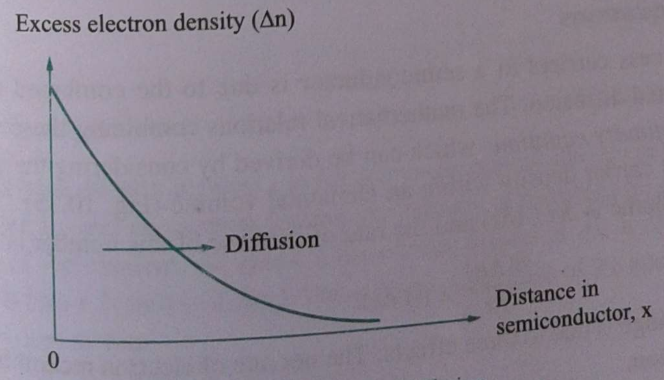


Fig. 10.34 Carrier diffusion

The force F on excess carriers restoring equilibrium is given by the product of excess charge and electric field, thus

$$F = (\Delta n)eE = [eD_n/\mu_n]\,\frac{\partial(\Delta n)}{\partial x} \qquad (10.68)$$

This force also depends on the thermal energy of the excess carriers. By making an analogy between the excess carriers in a semiconductor and gas molecules in a low pressure gas, the force corresponding to the pressure gradient is equal to $(k_BT)\,\dfrac{\partial}{\partial x}\,(\Delta n)$, as from the kinetic theory gases, gas pressure is equal to nk_BT where n is the molecular concentration. Comparing this force term with equation (10.68) gives

$$k_BT = eD_n/\mu_n$$

i.e.,

$$D_n = \mu_n k_B T/e \qquad (10.69)$$

The relation between diffusion coefficient and mobility of a charge carrier is termed *Einstein's relation*:

Now

$$\boxed{\begin{aligned}\frac{D_n}{\mu_n} &= \frac{k_BT}{e}\\ \frac{D_p}{\mu_p} &= \frac{k_BT}{e}\\ \frac{D_n}{D_p} &= \frac{\mu_n}{\mu_p}\end{aligned}} \qquad (10.70)$$

and

Hence

At $T = 300$ K, $k_BT/e = 26$ mV. Using this relation the electron and hole diffusion coefficients for various semiconductors can be calculated from the mobility.

The values for silicon are $D_n = 3.6 \times 10^{-3}\ \text{m}^2/\text{s}$ and $D_p = 1.3 \times 10^{-3}\ \text{m}^2/\text{s}$ while those for germanium are

$$D_n = 0.01\ \text{m}^2/\text{s} \quad \text{and} \quad D_p = 5 \times 10^{-3}\ \text{m}^2/\text{s}.$$

XIII. GENERAL STUDY OF EXCESS CARRIER MOVEMENT

The Continuity Equations

The behaviour of excess carriers in a semiconductor is due to the combined effects of generation, recombination, drift and diffusion. The mathematical relations combining these effects are termed the electron and hole *continuity equations* which can be derived by considering the combined time rate of change of the excess carrier density within an elemental volume (Fig. 10.35). The number of excess electrons in such a volume is $\Delta n\ (A\delta x)$ and the rate of increase of this number is

$$\frac{\partial(\Delta n)}{\Delta t} \times (A\ \delta x)$$

This rate of change is due to three effects: The net rate of electron recombination and movement due to drift and diffusion.

Net Recombination Rate

Equation (10.54*a*) is a first order representation of the net rate of increase of the number of excess of electrons in the volume due to the difference between th